Double Light Speed: History, Confusion and Recent Applications Working Paper in Progress! Not finished yet! Comments are welcome!

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Abstract

Every physicists and most non-physicists have heard that no object or signal can move faster than the speed of light in a vacuum c. When carefully studying the literature on Einstein's special relativity theory, one can also find a less known $2c$ speed limit (double light speed limit) that is fully consistent with special relativity theory. In the present piece we will first give a chronological historical overview of the literature on the $2c$ speed limit and then show how this speed limit has recently started to play a potential role in financial markets via high-speed trading.

Key words: Special relativity theory, speed limit, Einstein's velocity addition theorem, Lorentz velocity transformation, Galilean velocity addition, mutual velocity, exterior relative velocity, closing speed, high-speed trading.

1 The Einstein Velocity Addition Theorem

Almost every book on special relativity mentions Einstein's 1905 velocity addition theorem. Assume a train moving at velocity $0.5c$ as measured from the embankment emits a beam of light in the opposite direction of the train's direction relative to the embankment. From the train and relative to the train, the speed of light will be c as measured with Einstein synchronized clocks. Einstein correctly postulated that the speed of light relative to the embankment must adhere to the following velocity addition formula:

$$w = \frac{v+u}{1+\frac{vu}{c^2}}.$$

For example, if the train's speed is $v = 0.5c$ and the speed of light in a vacuum as measured from the train is $u = c$, then we find that the speed of light relative to the embankment as observed from the embankment via Einstein synchronized clocks must be

$$w = \frac{v+c}{1+\frac{vc}{c^2}} = c.$$

What the Einstein velocity addition theorem really shows is that the speed of light relative to a frame is independent of the speed of the light's source. The Einstein velocity addition theorem is naturally fully consistent with Einstein's second postulate about the constancy of the speed of light in any reference frame. In several older texts the velocity addition formula is referred to as a composition velocity or composite velocity (see for example [30] and [2]). The term *composite velocity* is appropriate, as the Einstein formula involves combining the velocity as measured from one frame with the velocity as measured from another frame and then transforming this composite velocity into the velocity relative to the observer frame.

The Einstein velocity addition formula can be derived directly from the Lorentz transformation and is often also called the *Lorentz velocity transformation* (see for example [19] and [9]). Actually, as pointed out by a series of texts, the Galilean velocity addition rule is only a good approximation to the Einstein velocity addition formula when v and u are much smaller than the speed of light in a vacuum.

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What Einstein rarely discusses is the speed of light relative to the train as observed from the embankment. This velocity is only indirectly discussed by Einstein in his discussion of relativity of simultaneity in his book *Relativity, The Special and General Relativity Theory*[1], where Einstein seems to acknowledge that the speed of light relative to a moving frame as observed from another frame is not c, but c plus or minus the observed speed of the moving object. In this book, when referring to a train moving across a railway, Einstein says,

> *When we say the lightning strokes A and B are simultaneous with respect to the embankment, we mean: the rays of light emitted at the places A and B, where the lightning occurs, meet each other at the mid-point M of the length A to B of the embankment. But the events A and B also respond to position A and B on the train. Let M' be the mid-point of the distance A to B on the traveling train. Just when the flashes of lightning occur, this point M' naturally coincides with the point M, but it moves towards right in the diagram with the velocity v of the train. If an observer sitting in the position M' in the train did not possess this velocity, then he would remain permanently at M, and the light rays emitted by the flashes of lightning A and B would reach him simultaneously, i.e. they would meet just where he is situated. Now in reality (considered with reference to the railway embankment) he is hastening towards the beam of light coming from B, whilst he is riding on ahead of the beam of light coming from A. Hence the observer will see the beam of light emitted from B earlier than he will see that emitted from A. Observers who take the railway train as the reference-body must therefore come to the conclusion that the lightning flash B took place earlier than the lightning flash A.* – Albert Einstein 1916

Pay attention to Einstein's sentence "*Now in reality (considered with reference to the railway embankment) he is hastening towards the beam of light coming from B, whilst he is riding on ahead of the beam of light coming from A*". Here Einstein indirectly points out that the speed of light relative to a moving frame (in this case, a train) as observed from another frame is not c. The speed of light relative to the train as measured on the train is indeed c, as is the speed of light relative the embankment. As we will soon discuss in a historical perspective, the speed of light relative to a moving frame as observed from another frame actually follows the Galilean addition rule.

Several textbooks covering special relativity give the incorrect impression that the Galilean velocity addition rule cannot at all be used under special relativity for objects or signals moving at speeds close to c. Very few texts discuss the speed of light relative to the train as observed from the embankment, or the speed of light relative to light as observed from a frame that itself is moving slower than c. This is where the Galilean addition rule applies and where we have a double light speed limit of $2c$ rather than c.

2 The History of the $2c$ Speed Limit

The literature review entailed an attempt to examine every book published since 1905 (every book in English and also some books in German) on special relativity theory. The books were then searched to see if they commented on double light speed.[2] Papers were much more difficult since thousands of papers are published on special relativity theory. To economize my efforts, I relied in internet search tools and consulted reference lists in the few sources I found on this topic. While all of the books point out the speed limit of light c in Einstein's velocity addition formula, there seems to be far less space given to discussing the $2c$ speed limit. Furthermore, we could only find a couple of published paper briefly elaborating on the $2c$ speed limit. The little attention that has been paid to the $2c$ speed limit has come from a series of quite well-known physicist supporting Einstein's special relativity theory.

The first source, from the 1911 book *Das Relativitatsprinzip* by German physicist Max von Laue (who received the Nobel Prize in physics in 1914), tells us that the relative velocity between two objects (or even light) can be added together by classical Galilean velocity addition rules if observed from a frame that is separate from the frame in which the two objects are located. Yet the fact that this velocity follows classical velocity addition rules is only mentioned in a footnote under his section on Einstein's velocity addition formula:

> It should be noted that the rate of speed q is based on a different system than v. If both are to refer to the same system, it follows that ordinary vector addition must be applied. For example is the speed of light against a moving rod, based on a not co-moving system, still the vector sum of the speed against the system and the speed of the rod....[3]

[1] I am referring to the 1961 Robert W. Lawson translation of Einstein's 1916 book

[2] It is possible that I missed out on a few books, but I think I have covered most of them.

[3] See [20] page 4344.

Max von Laue does not mention that the speed limit for this type of velocity is $2c$, but when familiar with the subject, it is quite clear that the limit will follow from his statement: "*If both are to refer to the same system, it follows that ordinary vector addition must be applied* ".

In his 1914 book *The Theory of Relativity* Polish-American physicist Ludwik Silverstein, in a footnote under the section on composite velocities, basically says the same as Max von Laue:

> If both were taken with respect to the same system, then their resultant would, of course, be simply equal to their vector sum. But this is hardly worth mentioning. For all cases of composition of velocities, which have any physical interest, are of the considered above, viz. imply component velocities to a chain of different systems: An object B moves in a given way relative to A, a third object C moves relative to B, and so on; find the motion of the last relative to the first.[4]

The first source directly mentioning the double light speed limit[5] seems to be a 1948 book published by the German physicist Arnold Sommerfeld. In his book Sommerfeld states that[6]

> To begin with, we obviously mean by "velocity" "relative velocity". But that does not suffice. Consider a sample of radium. It emits electrons with almost the velocity of light. Two electrons which fly off simultaneously in opposite directions have a very nearly the relative velocity $2c$, viewed from the laboratory in which the sample of radium is at rest. However, in order to properly define relative velocity as used in our statement we must view one electron from the other. Then and then only the seemingly paradoxical equation $c + c = c$ applies. We are thus concerned, in our statement, with the relative velocity of a moving point with respect to a reference system which is transformed to a state of rest.

In 1954 science philosopher Adolf Grünbaum published a paper [11] titled *The Clock Paradox in Relation to Special Relativity Theory*, where he correctly applied standard Galilean velocity addition rules for an observer of two other observers moving relative to him. We are not going to discuss the clock paradox here, but in 1955 physicist Boris Leaf follows up in the same journal [23] and criticizes Grünbaum for not having used the Einstein velocity addition formula:

> One of Grünbaum's observers, the one at O in K, it is claimed, judges U_3 approaches A at a velocity of $V - v$.Such a velocity addition is not in accord with the Einstein formula. Since the judgment of this observer is a vital link in Grübaud's argument, I am unable to accept his treatment as valid.

Grünbaum [12] replies by pointing out that

> ... relativity theory asserts that no signal or influence chain can travel faster than the velocity c of light. But a velocity of separation of two bodies in K is not the velocity of a signal, there being no *direction* of propagation (and no point of origin generally). Thus, even if we compound vectroically a velocity $V = .9c$ and a velocity $v = -.3c$, both relative to K, to obtain a velocity of separation $V - v = 1.2c$ of the two bodies *relative* to K [but NOT relative to one another!], this does not contradict the relativity theory at all.

In his 1959 book on the theory of special relativity, physicist Joseph Aharoni writes that

> Composition of velocities of transformation of a velocity from one inertial frame to another must be distinguished from adding velocities which are all defined relative to a given frame. Thus relative to K two bodies move along the x-axis with velocities v_1 and v_2 respectively, then relative to K the velocity between these two bodies is $v_1 \pm v_2$ depending on whether they move in opposite directions or in the same direction. In particular, relative to K the velocity between two light signals traveling in opposite direction is $2c$. Each of the light signals covers in one second the distance c and the distance between the light signals after one second is therefore $2c$.[7]

A year later, in his book titled *Special Relativity*, physicist Wolfgang Rindler writes,

> Velocities measured in the *same* frame are, of course, compounded as in the classical theory according to the usual vector rule. Thus the relative velocity of two particles as measured in a frame in which both move can be as much as $2c$.[8]

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[4] See [30] page 164.

[5] While we have reviewed a lot of the literature in special relativity, we could have overlooked texts that mentioned this even earlier. But our attempt at finding the first source describing $2c$ was systematic and based on our expertise within the field.

[6] The quotation is from page 231 of the 1952 English translation.

[7] See [2] page 18.

[8] See [26] page 36.

Percy Bridgman, who won the 1946 Nobel Prize in Physics for his work on high pressures, published a book in 1962 titled *A Sophisticate's Primer of Relativity.* His book is in support of Einstein's special relativity theory and at the same time tries to discuss the theory in depth. Below is a quote from Bridgman:

> In a single system, the upper limit for the relative velocity of two beams of light or of two material objects is not c but $2c$.[9]

Bridgman further states that

> Of course, if both velocities were measured in the same frame, the classical simple addition formula for relative velocities would continue to hold. In particular, in the stationary frame the velocity of a particle moving to the left with a velocity of $0.75c$ relative to a particle moving to the right with a velocity $0.75c$ is $1.5c$, as it always has been and always will be, despite frequent statements that relative velocities higher than c do not occur.

On Einstein's velocity addition formula, Bridgman also points out that

> This formula has been the occasion of much unnecessary misunderstanding and paradox.[10]

Not much has changed since then; there still seems to be quite some confusion among many physicist on this point. And most books do not even mention that the speed of an object against another object as observed from a third frame follows Galilean velocity addition rules and can have relative velocities up to $2c$, which is fully consistent with special relativity. In his 1965 book, theoretical physicist Huseyin Yilmaz writes the following:

> At this point, we may also point out that no *relative velocity* can exceed c, for the Lorentz transformations would be imaginary. But this does not mean that no velocity higher than c can be considered. For example, take two trains approaching a station from opposite directions, both with the velocity of light. According to an observer at the station, the two trains are approaching each other with a velocity of $2c$ (here two objects are in questioning). But with respect to a man on one of the trains, the other train is approaching with a velocity of $w = (c + c)/(1 + c^2/c^2) = c$.[11]

Yilmaz never accepted the general theory of relativity and tried to come up with his own gravitational theory, known as the Yilmaz theory of gravitation. But the quote above is related to special relativity theory, which he fully supported.

When compared to his 1960 book, Wolfgang Rindler's 1969 book contains a more detailed elaboration of the $2c$ speed limit:

> We may note that, relative to any frame S, two particles or photons may have a *mutual velocity* up to $2c$. This velocity is the time rate of change if the connecting vector $r_2 - r_1$ between the particles, which we assume to have position vectors and velocities r_1, u_1 and r_2, u_2, respectively: $(d/dt)(r_2 - r_1) + u_2 - u_1$, as in classical kinematics. For example, the mutual velocity of two photons traveling in opposite directions along a common line is precisely $2c$.[12]

We can see that Rindler is now introducing the term *mutual velocity* to distinguish this velocity from velocities where the Einstein velocity addition formula is needed. No other texts appear to have adopted his terminology.

In 1977 at the meeting of the Tamil Nadu Academy of Sciences[13] Alladi Ramakrishnan presented a short note titled "A New Concept in Special Relativity Exterior Relativity Velocity", where he wrote the following:

> We shall introduce a new concept in special relativity called "the exterior relative velocity", which satisfies the Newtonian addition law but is consistent with and in fact derived from Einstein's law of addition of relative velocities.

Reading the whole text reveals that this is exactly the same velocity that [20], [31], [11], [2], [26], [4], and [34] had already described years before. Ramakrishnan is clearly unaware that a few physicist had already mentioned this. Yet according to his own account, he studied special relativity for many years. With only a few previous books and only one academic paper mentioning the $2c$ speed limit, it is no wonder why Ramakrishnan was unaware of others having published on it. In 1997 Ramakrishnan

[9] See [4] page 108.
[10] See [4] page 107.
[11] See [34] page 30.
[12] See [27] page 36. See also the updated versions [28, 29] that basically state the same as his 1969 edition on this topic.
[13] See also [25].

published a paper in the *Journal of Mathematical Analysis and Applications* where he again mentioned *exterior relative velocity* and that this can take values up to $2c$ and still be fully consistent with special relativity theory. This is the second academic paper we have found discussing the $2c$ speed limit in relation to special relativity theory. Velocities where Einstein's velocity addition formula is needed are what Ramakrishnan calls *internal relative velocities.* Ramakrishnan points out that interior relative velocities can take speeds of up to c, while exterior relative velocities can take speeds of up to $2c$, and both are fully consistent with special relativity theory.

Oxford physicist Michael Bowler, in his 1986 book, briefly mentions the double light speed limit:

> In a colliding beam experiment e^+ and e^- are circulating in opposite directions, each with energy 15 GeV. What is their relative velocity in the laboratory? The answer is marginally less than $2c$, the sum of the two laboratory velocities. But the velocity of e^+ in the e^- rest frame is still marginally less than c, and vice versa, and again there is no question of signals being propagated faster than light. [14]

In his 2002 book, physics Professor Moses Fayngold writes:

> One could ask: "I see two objects flying apart each with a speed $0.9c$ relative to me, don't I therefore see the flying apart with relative speed $1.8c$? The answer to this would be "No". They do fly apart with speed $1.8c$, but this is not the speed of their *relative* motion. The relative speed is observed in the rest frame of one of the objects.

In his 2008 book Jürgen Freund discusses the double light speed limit in relation to the Einstein velocity addition theorem:

> This is the place to draw the reader's attention to a likely misunderstanding of the addition theorem of velocities: it may only be used *if the velocity on the left-hand side and the velocity on the rich-hand side are measured in different reference frames. If two velocities* u_{x1} *and* u_{x2} *are measured in the same reference frame, and one wants to calculate their velocity difference, the formula is* $\Delta u_x = u_{x1} - u_{x2}$, *even if the velocities are relativistic. As an example, let us think of two photons traveling at* $+c$ *and* $-c$ *along a line, their velocity difference is, of course,* $2c$. *From the viewpoint of one photon, however, the other photon's velocity is c...* [15]

Some four years later, in 2012, Andrew Steane, Oxford professor in theoretical physics, writes that,

> To distinguish the two ideas, sometimes $u+v$ is called a "closing speed" while w is a relative speed. A closing speed can be greater than the speed of light (up to a maximum $2c$), but this does not break the Light Speed Postulate because no signal is traveling at $u+v$.[16]

As we can see Steane here introduces yet another term, namely *closing speed*, to distinguish it from velocities where the Einstein velocity addition theorem is needed.

Among more standard university textbooks introducing special relativity, only [14, 15] seems to comment on the *double light speed limit*:

> A bit of classical physics should be noted. The meteorite's speed relative to the spaceship is indeed $1.4c$, according to an observer on Earth. If Bon on Earth sees a spaceship moving away at $0.8c = 2.4 \times 10^8$ m/s and the meteorite is moving towards Earth at $0.6c = 1.8 \times 10^8$ m/s, the two are surely getting 4.2×10^8 m closer every second. This logic is sound because all three of these velocities are according to the same (Earth) observer. To calculate such a quantity, the relativistic velocity transformation is not appropriate, for it rates velocities according to different observers. But the $1.4c$ is not an example of something moving relative to an observer at a speed greater than c. It is the meteorite's speed according to the Earth observer but relative to the space-ship. Because in special relativity we are not usually interested in a velocity between two objects according to a third party, when we say simply "the velocity of B relative to A," it is understood to mean according to A.[17]

Other books that briefly touch upon the relative velocities between two objects as observed from an external frame are [16], [13], [22], [1], [18], [8] and [6].

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[14] See [3] page 60.
[15] See [10] page 84.
[16] See [32] page 140.
[17] See [15] page 32.

3 Lack of Standardized Terminology

From the section above we can discern a lack of standardized terminology. The velocity between two objects (or even light) as observed from a third frame is called *velocity of separation* by Grünbaum, also referred to as *relative velocity* by Bridgman. Rindler called it *relative velocity* in 1960, and then in 1969 he introduced the term *mutual velocity* to distinguish it from velocities where Einstein's velocity addition formula is needed. Ramakrishnan coined yet another term, *exterior relativity velocity*, and Steane introduced the term *closing speed*. Yet Fayngold argues that relative velocity is for velocities between two objects where one of the objects is an observer.

Physicist seem to understand the physics and math involved here; the challenge is mainly a lack of standardized terminology. That some physicists call it *relative velocity* while others do not provides little help for clarifying for readers who are familiar with only a few books on special relativity theory. This is especially true for physics students who often have just a few chapters on special relativity during their whole university study.

It is not surprising that many physicists who have not studied special relativity theory very carefully seem confused on this topic. Most likely they have followed one of the many textbooks that fail to mention the existence of a $2c$ speed limit that is fully consistent with special relativity. Others have heard of the $2c$ speed limit, but the lack of standardized terminology still seems to add to the confusion. Should it be called relative velocity, mutual velocity, exterior relative velocity or closing speed? Furthermore, there seems to only be a couple of academic papers mentioning the double light speed limit.

In my view there is nothing wrong in calling this velocity *relative velocity*. All velocities are of something relative to another thing. It seems more correct to return to the term composite velocities is cases where the Einstein velocity formula is needed, rather than claiming that the velocity of a train relative to another train as observed from the embankment not is a relative velocity. It is also incorrect to state that this velocity with a limit of $2c$ cannot be used to carry signals. As we soon will discuss, the double light speed limit is exactly what is explored by financial firms that trade from midpoint colocations. This is a real velocity that is used to carry crucial information. Rather than deciding on what this velocity should or should not be called, physicists should be more careful when they talk about and define velocities: What is the velocity measured relative to, and from what frame is this velocity observed?

4 Recent Applications in High-Speed Trading

In finance there has always been a race to minimize the time it takes to obtain information. The last few years hundreds of millions of dollars have been spent by setting up microwave towers between Chicago and New York as well as between other financial centers [21]. The speed of light in air is about $0.997c$, so we are truly approaching the speed of light in high-speed trading. Hundreds of millions of dollar have also been used to straighten fiber-optic cables between financial centers to minimize the distance the information needs to travel.

Colocations play an important role in high-speed trading. "If you want to trade stocks *within* the New York Stock Exchange, then you want to be as close as possible to the exchange. You can place your computers and trading algorithms in a so-called exchange colocation. Most exchanges today offer exchange colocation. This is basically a space for computers to sit next to the matching engine. Still, if you want to compare information from different marketplaces, for example the price of a company trading both in New York and Chicago If you have a single colocation in New York or in Chicago, then the fastest method for comparing prices is by the speed of light c. Typically, you want to compare the prices and then give an execution order to both exchanges. You need to cover the distance between the marketplaces twice, and the fastest way to do this if you have a single exchange colocation is naturally with the speed of light. The shortest time this can take is

$$\frac{2d}{c},$$

where d is the distance between the two marketplaces. However, by placing your trading algorithms (computers) in the middle between the two exchanges in a so-called midpoint colocation, you will take advantage of the $2c$ speed limit. You are then using two light signals, and the speed of light against light as observed from the ground is $2c$. The time to compare and execute on both exchanges is now

$$\frac{2d}{2c}.$$

Very little has been written on midpoint colocation. The first paper discussing the advantage of midpoint colocation seems to be [33]. It points out that midpoint colocation will be advantageous. A recent article in Nature [5] also mentions midpoint colocation:

> In the future, when airborne laser networks span the oceans, things may get even stranger. The location at which traders get the earliest possible information from two exchanges lies at their midpoint between Chicago and London, this is in the middle of the Atlantic Ocean. At such a site, traders could exploit a technique called relativistic arbitrage' to profit from momentary imbalances in prices in Chicago and London.

Dismissing the idea of benefits from midpoint colocation, Bill Harts, CEO of Modern Markets Initiative, a group supporting high-frequency trading, told MarketWatch that

> Even if a trader could receive data from a market faster at such a location, in order to profit from it she would have to transmit orders to a market from the same location and would always be slower than traders stationed at the actual markets.

Where there is disagreement, there is often confusion. As first pointed out by [17] we have three main categories of colocation set-ups for high speed trading. First, let us consider a trader in an exchange colocation that is only focusing on local trading. With local trading I am thinking only of comparing assets that are trading inside the same exchange. The trader can compare securities listed on the same exchange almost instantaneously (or at least extremely fast compared to someone located outside the exchange). Assume that the distance between two exchanges is d. The trader set-up in this case will use $t = \frac{d}{c}$ to compare the price of the assets on the two exchanges. Then the trader will use $\frac{2d}{c} = 2t$ to compare and execute on both exchanges.

Next assume a trader who only has a midpoint colocation. For the midpoint colocation, comparing assets on the same exchange and executing the trades will take $\frac{0.5\times d}{c} + \frac{0.5\times d}{c} = \frac{d}{c} = t$. Clearly the exchange colocation is superior for comparing assets on the same exchange (local trading). When it comes to comparing the price of assets on two exchanges, the midpoint colocation uses $\frac{d}{2c} = \frac{1}{2}t$. to compare the prices. In addition, to execute trades on that information, the midpoint colocation uses $\frac{2d}{2c} = t$. So a midpoint colocation is clearly superior to a single exchange colocation for such trading; actually it is twice as fast, given t versus 2t. This is due to the fact that the speed of light relative to light as observed from a third frame (the ground in this case) is $2c$.

However, we can have a third type of set-up, where we have exchange colocations at each exchange. When dealing with two exchanges, we can call this set-up "twin exchange colocation." From these twin colocations, local trading can naturally be done almost instantaneously. To compare assets from the two exchanges takes now $\frac{d}{c} = t$ and to execute trades based on this information basically will not add any time (when, for simplicity sakes, we assume that the distance from the exchange colocation to the exchange is 0). The table below summarizes the three trading strategies:

LATENCY TABLE	Exchange cl	Midpoint cl	Twin exchange cl
Local trading:	≈ 0 $(0/c)$	t (d/c)	≈ 0 $(0/c)$
Compare:	t (d/c)	$0.5t$ $(d/2c)$	t (d/c)
Compare-execute:	$2t$ $(2d/c)$	t $(2d/2c)$	t (d/c)
Compare-execute-compare:	$3t$ $(3d/c)$	$1.5t$ $(3d/2c)$	$2t$ $(2d/c)$
Compare-execute-compare-execute:	$4t$ $(4d/c)$	$2t$ $(4d/2c)$	$2t$ $(2d/c)$

Table 1: Except for line one, the other lines have to do with what happens when we are comparing prices between two exchanges. Only the lines in bold are of real importance; for trading it does not help to get information first if you are not taking into account the time it takes to act (execute) on that information.

Most firms involved in statistical arbitrage between exchanges will also be involved in local trading strategies at each exchange. In general, this means that they do not need midpoint colocation. It could however be that midpoint colocation has cheaper rental space than exchange colocation. Firms that are only involved in arbitrage trading between two exchanges could prefer midpoint colocation for this reason. There could also be some exchanges that do not offer exchange colocation. It is not necessarily that cheap to buy or get access to the building next door to the exchange (to use for colocation), so in such situations midpoint colocation could potentially be superior for arbitrage trading between exchanges, as compared to nearby exchange colocations. Having a colocation building close to the exchange do not ensure the communication path is short see [24] for examples on this.

5 Conclusion

The $2c$ speed limit seems to have been first mentioned directly by Sommerfeld in 1948, and has later been mentioned in several books and papers on special relativity theory. Yet the percentage of books even mentioning this speed limit is likely less than 5%. There is also a lack of standardized terminology for the speed of an object (or light) relative to another object as observed from a third frame. Bridgman called it relative velocity, while Rindler tried to introduce the term *mutual velocity* to distinguish it from velocities where Einstein's velocity addition theorem is applicable. Ramakrishnan called it *exterior relative velocity*, and Steane called it*closing speed*. The lack of standardized terminology, combined with the fact that most university text books do not even mention a $2c$ speed limit that follows Galilean velocity addition rule, is likely the reason why there still is confusion around this topic. It is important to be aware of the two speed limits in special relativity theory. The maximum speed of light relative to an observer as observed by the same observer is c. The maximum speed between two objects as observed from a third frame is $2c$. The speed of light against light (in vacuum) as observed from any other frame than light itself is $2c$.

The double light speed limit has in recent years started to play a potential role in high-speed trading, where one compares assets between two marketplaces. Midpoint colocations can not outcompete twin exchange colocation, but can at least in theory compete with exchange colocations when only focusing on arbitrage trading between exchanges.

References

[1] S. Adams. *Relativity: An Introduction to SpaceTime Physics*. Taylor and Francis, 1997.

[2] J. Aharoni. *The Special Theory of Relativity*. Oxford at the Clarendon Press, 1959.

[3] M. G. Bowler. *Lectures on Special Relativity*. Pergamon Press, 1986.

[4] P. W. Bridgman. *A Sophisticate's Primer of Relativity*. Dover Publications, 1962.

[5] M. Buchanan. Physics in finance: Trading at the speed of light. *Nature*, 158:162–163, 2015.

[6] S. Catalano. *The Theory of Relativity, From Notes to Myself*. Self Published, 2003.

[7] A. Einstein. On the electrodynamics of moving bodies. *Annalen der Physik, English translation by George Barker Jeffery 1923*, (17), 1905.

[8] M. Fayngold. *Special Relativity and Motions Faster than Light*. Wiley, 2002.

[9] R. A. Freedman and H. D. Young. *University Physics with Modern Physics, 14th Edition*. Pearson, 2016.

[10] J. Freund. *Special Relativity for Beginners, A Textbook for Undergraduates*. World Scientific, 2008.

[11] A. Grünbaum. The clock paradox in special relativity theory. *Philosophy of Science*, 22:249–253, 1954.

[12] A. Grünbaum. Reply to dr. leaf. *Philosophy of Science*, 22:53, 1955.

[13] A. Grünbaum. *Philosophical Problems of Space and Time*. Springer, 1973.

[14] R. Harris. *Nonclassical Physics, Beyond Newton's View*. Addison Wesley, 1999.

[15] R. Harris. *Modern Physics, Second Edition*. Pearson, 2008.

[16] T. M. Helliwell. *Introduction to Special Relativity*. Allyn and Bacon, 1966.

[17] E. Howorka. Colocation beats the speed of light. *Article/working paper that can be found at www.edhoworka.com*, 2015.

[18] J. B. Kogut. *Introduction to Relativity*. Harcourt Academic Press, 2000.

[19] K. Krane. *Modern Physics, Third Edition*. John Wiley & Sons, 2012.

[20] M. V. Laue. *Das Relativitatsprinzip*. Braunschweig, 1911.

[21] G. Laughlin, A. Aguirre, and J. Grundfest. Information transmission between financial markets in chicago and new york. *arXiv.org*, 2013.

[22] D. F. Lawden. *Elements of Relativity Theory*. John Wiley & Sons, 1986.

[23] B. Leaf. The clock paradox in special relativity theory. *Philosophy of Science*, 22:45–52, 1955.

[24] M. Lewis. *Flash Boys: A Wall Street Revolt.* W. W. Norton & Company, 2015.

[25] A. Ramakrishnan. A new concept in special relativity—exterior relative velocity. *Proceedings Tamil Nadu Academy of Sciences*, 1:67–69, 1978.

[26] W. Rindler. *Special Relativity.* Oliver and Boyd, 1960.

[27] W. Rindler. *Essential Relativity, Special, General and Cosmology.* Springer Verlag, 1969.

[28] W. Rindler. *Relativity, Special, General and Cosmology.* Oxford University Press, 2001.

[29] W. Rindler. *Relativity, Special, General and Cosmology, Second Edition.* Oxford University Press, 2006.

[30] L. Silvertstein. *The Theory of Relativity.* MacMillian, 1914.

[31] A. Sommerfeld. *Elekrodynamik, Vorlesnungen über Theoretische Physik: Band III.* Dieterich, 1948.

[32] A. Steane. *The Wonderful World of Relativity: A Precise Guide for the General Reader.* Oxford University Press, 2012.

[33] A. D. Wissner-Gross and C. E. Freer. Relativistic statistical arbitrage. *Physical Review E*, 82, 2010.

[34] H. Yilmaz. *Introduction to the Theory of Relativity and the Principles of Modern Physics.* Blaisdell Publishing Company, 1965.